

Sections 8.2-8.4 part 4

Every normal subgroup is the kernel of a homomorphism } The kernel of homomorphism  
 Th 8.18 Let  $N$  be a normal subgroup of a group  $G$ . } is the ideal  $I$

The map

$$\pi: G \rightarrow G/N$$

$$g \mapsto Ng \quad \left\{ \begin{array}{l} Ng = gN \\ \end{array} \right.$$

is a surjective homomorphism, and  $N$  is its kernel.

Pf ① the map is a homom:  $\pi(a) = Na \quad \pi(b) = Nb \quad \pi(ab) = Nab$

$$\text{Wanted: } \pi(a)\pi(b) = \pi(ab)$$

$$\pi(a)\pi(b) = NaNb = N(ab) = \pi(ab)$$

②  $N = \ker(\pi)$ :  $\ker(\pi) = \{a \in G \mid \pi(a) = N\}$  }  $N = Ne_G$ , the identity in  $G/N$

$$\pi(a) = N \text{ means } Na = N \text{ means } a \in N$$

③ surjective

for every  $Na \in G/N$ , we have  $\pi(a) = Na$ .

Th 8.20 First Isomorphism Thm

Let  $f: G \rightarrow H$  be surjective group homomorphism  
with a kernel  $K = \ker(f)$ .

Then  $G/K \cong H$ .

Pf Let  $\varphi: G/K \rightarrow H$   
 $ka \mapsto f(a)$

$$\varphi(ka) = f(a)$$

The definition of  $\varphi$   
depends on a choice made.  
a is  $\underline{\text{a}}$  representative of the  
coset  $ka$ .  $\left\{ \begin{array}{l} ka = kb \\ \text{for any } b \in K \end{array} \right.$

The map  $\varphi$  is well-defined:

If  $ka = kb$  then  $f(a) = f(b)$

$b \in Ka$   $b = ka$  with  $k \in K$ .

$$\underline{f(b)} = f(ka) = f(k)f(a) = e_H f(a) = \underline{f(a)}$$

$$(ka)(kb) = k(ab)$$

$\varphi$  is a homomorphism:  $\varphi(ka)\varphi(kb) = \varphi(kab)$

$$\underline{\varphi(ka)\varphi(kb)} = f(a)f(b) = f(ab) = \underline{\varphi(kab)}$$

$g$  is surjective: For  $b \in H$  we find  $a \in G$  such that  $g(ka) = b$ .

As  $f$  is surjective, there exists  $a$  such that  $f(a) = b$ .

Then  $g(ka) = f(a) = b$